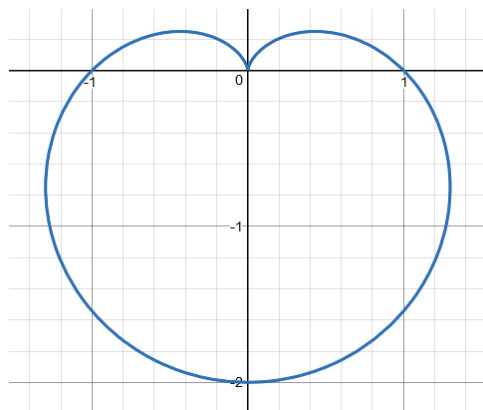
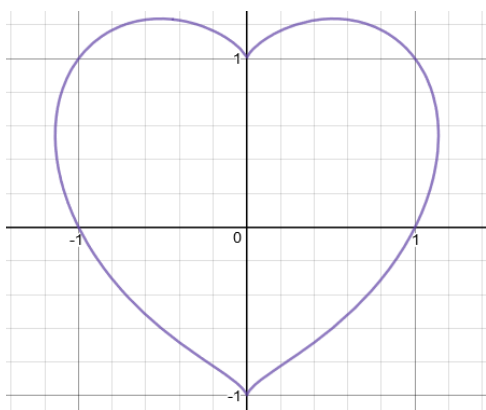


Quest for Mathematics I (E2): Exercise sheet 4

1. Find the equations of the tangent to the curves described by the following equations:

- (a) $y = x^3 - 2x + 1$ at the point $(1, 0)$;
- (b) $(x^2 + y^2 - 1)^3 - x^2y^3 = 0$ at the point $(1, 1)$;
- (c) $x(t) = \sin(t)(1 - \cos(t))$, $y(t) = \cos(t)(1 - \cos(t))$ at $(1, 0)$.

NB. Examples (b) and (c) are sketched as follows.



2. (a) Prove the mean value theorem for $f(x) = x^2$ directly, i.e. for $a < b$ given, identify a $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

(b) Use the mean value theorem to check that $e^x \geq 1 + x$ for all $x \in \mathbb{R}$.

3. After computing y' and y'' , sketch the following curves:

- (a) $y = \frac{(x-1)^2}{1+x^2}$, $x \in \mathbb{R}$;
- (b) $y = 1 + x - \cos(x)$, $x \in [0, 2\pi]$;
- (c) $y = x^{-1}e^x$, $x \in \mathbb{R}$;
- (d) $y = \log |\cos(x)|$, $x \in [-\pi/2, \pi/2]$.

In each case, you should identify and label (where relevant):

- asymptotes;
- stationary points (including whether they are maxima/minima/inflection points);
- which sections of the curve are increasing/decreasing;
- which sections of the curve are convex/concave.